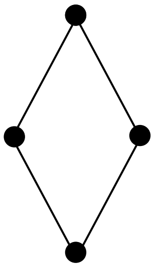
Name: Day:

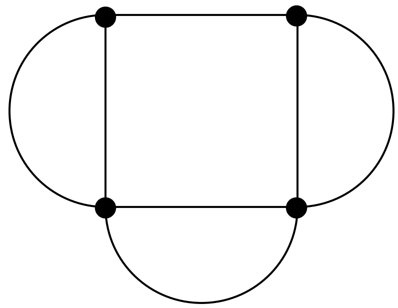
Networks and Paths

Try This: For each figure **A**, **B**, and **C**, draw a path that traces every line and curve exactly once, **without** lifting your pencil.

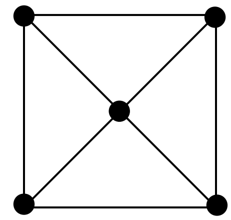
A.



B.



C.



Figures **A**, **B**, and **C** above are examples of **NETWORKS.**

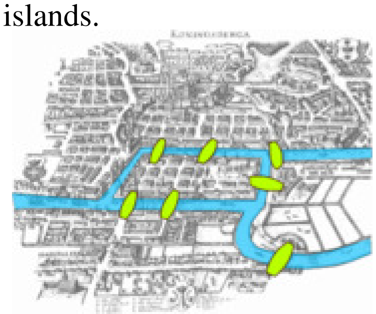
> A **network** is a set of connected line segments or curves.
> The line segments or curves that make up the network are called **edges.**
> The intersection of two or more edges is called a **vertex**.

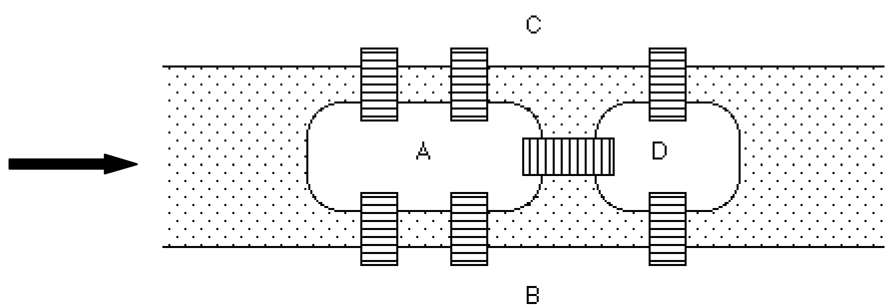
You should have been able to draw the required path for figures A, B, but not for figure C. Figures A and B are traversable networks, and figure C is an example of a non traversable network.

> A network is **traversable** if it can be drawn by tracing each edge exactly once, without lifting your pencil.

The study of networks in mathematics began in the middle 1700's with a famous puzzle called the "Seven Bridges of Konigsburg."

Konigsburg (now called Kalingrad and part of Russia) is a town with the Pregel River separating it into two parts. There are two islands in the Pregel River, with seven bridges connecting both parts of Konigsburg and the two islands.



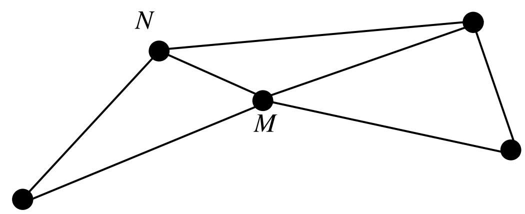


The challenge was to find a path that crosses each bridge exactly once. The students in town were unable to find such a path and finally asked Leonhard Euler, a famous mathematician, to help them. Using networks, Euler proved that the students could <u>not</u> find a path that crossed each bridge because there is none!
Euler discovered that it is possible to determine whether or not a network is traversable by examining its vertices and edges.

In a network, an edge is a straight or curved segment that connects two vertices. A vertex can be classified as either even or odd.

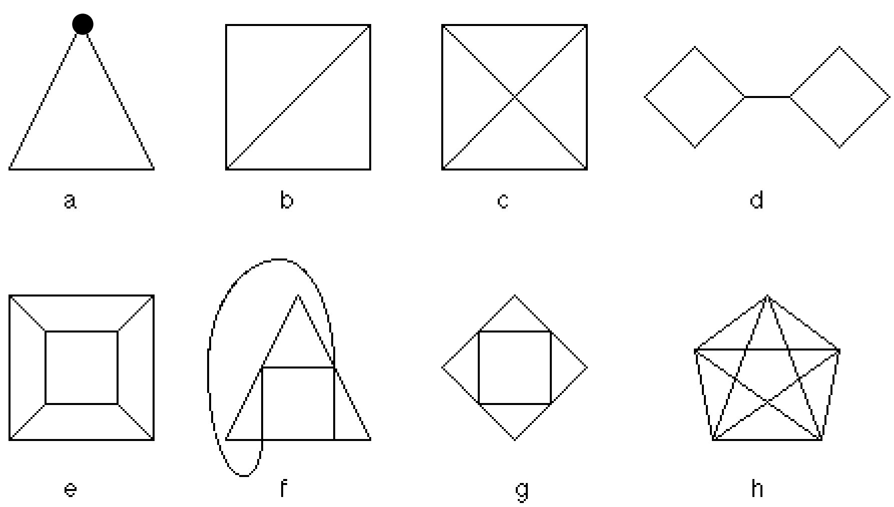
- A vertex is **even** if an even if an even number of edges meet at that vertex.
- A vertex is **odd** if an odd number of edges meet at that vertex.
-

M is an **even vertex** since 4 edges meet at M.
N is an **odd vertex since** 3 edges meet at N



In the following activity, you will see how Euler showed that there is no solution to the Konigsburg bridge problem. You will try to find a way to predict which networks are traversable and which networks are non traversable. You will also look for clues about the best place to start tracing.

- Place a point at each vertex. The first one is done for you.
- Circle all the odd vertices in the networks.
- For each network, count the number of odd vertices and the number of even vertices, then complete the table.



Network	# of even vertices	# of odd vertices	Traversable?
a			
b			
c			
d			
e			
f			
g			
h			

Using the information in the table above, answer the following questions:

1. Is a network with no odd vertices traversable?

2. Is a network with two odd vertices traversable?

   a. Did you start at an odd or even vertex?

   b. Did you finish at an odd or even vertex?

3. Is a network with more than two odd vertices traversable?

4. Based on your previous answers, how would you suggest we determine if a network is traversable?

5. Create several networks of your own. Does your method accurately predict whether they can be traced?

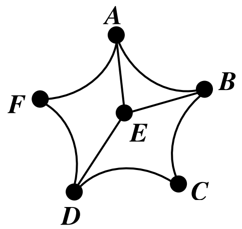
Euler made the following conclusion after studying the vertices of many networks:

A **network** will be traversable unless it has more than two odd vertices

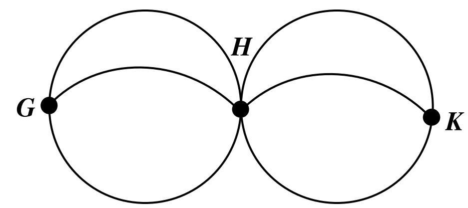
Exercises:

6. Tell whether each of the following networks is traversable. EXPLAIN how you arrived at your answer!

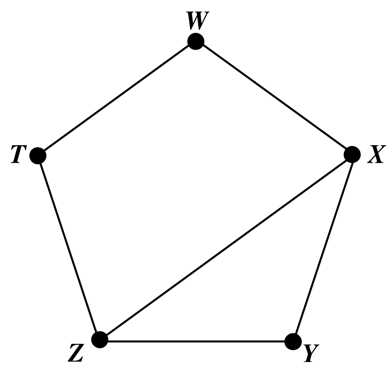
a.



b.



Use the following network to complete exercises 7 – 9.

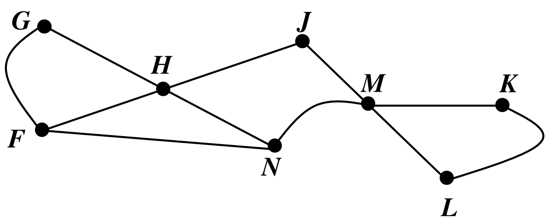


7. Without lifting your pencil, draw a path that traces each edge of the network exactly once. Name the vertices of your path in order.

8. Repeat exercise 7, beginning with a different vertex each time.

9. Name the vertices that **cannot** begin a path that traverses the network illustrated above.

Use the following network to complete exercises 10 – 15.



10. Without lifting your pencil, draw a path that traces each edge of the network exactly once. Name the vertices of your path in order.

___ ___ ___ ___ ___ ___ ___ ___

11. Name the vertices that **can** begin a successful path that traces each edge exactly once.

12. Name the vertices that **cannot** begin a successful path that traces each edge exactly once.

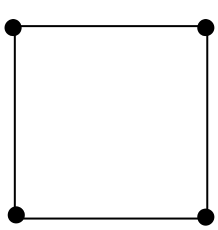
13. Classify each vertex from above as even or odd.

14. Write a prediction about the starting point of a successful path of a network having exactly two odd vertices.

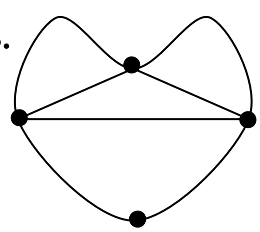
15. Draw another network having exactly two odd vertices and test the prediction that you made in exercise 14.

16. Four friends will compete against each other in a backgammon tournament. Each person will play the others exactly once. Which of the following networks may be used to illustrate the games in the tournament?

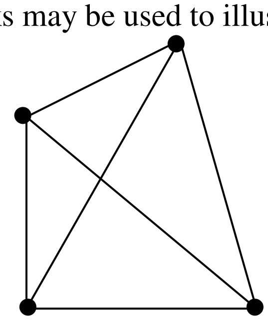
A.



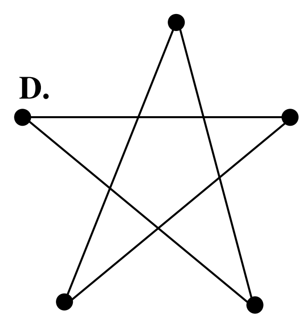
B.



C.



D.

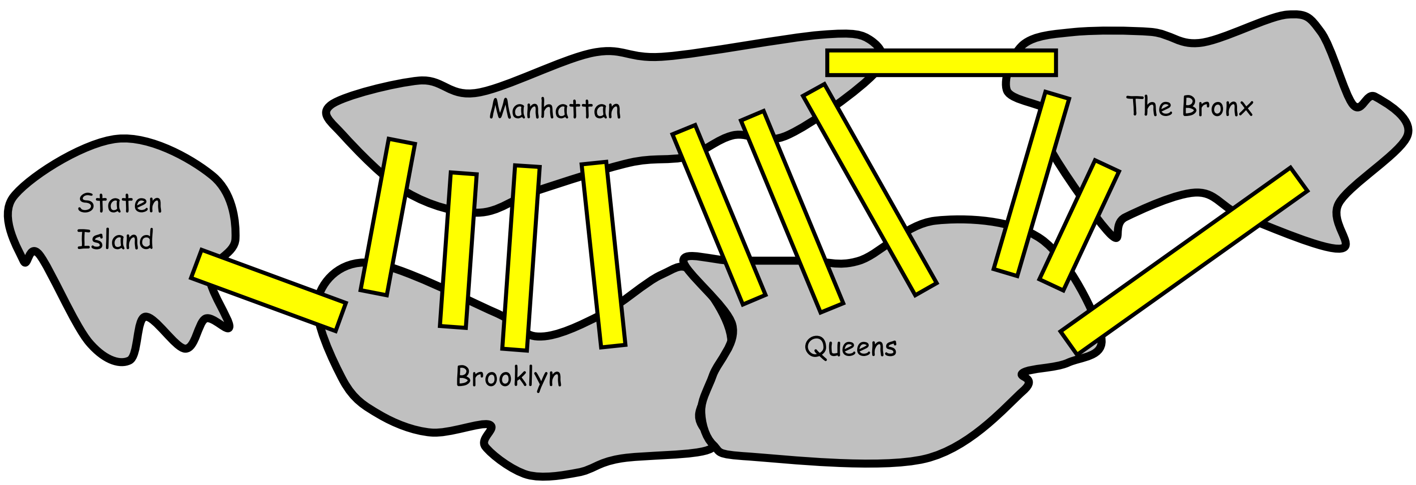


17. The principals of six different middle schools are meeting for the first time. Each principal must interview all of the other five principals individually to learn about interesting programs in his/her school. Draw a network that represents all of the meetings that must take place.

18. a. List five uppercase letters of the alphabet that are traversable. Pick one and explain why it is traversable.

b. List five uppercase letters of the alphabet that are **not** traversable. Pick one and explain why it is not traversable.

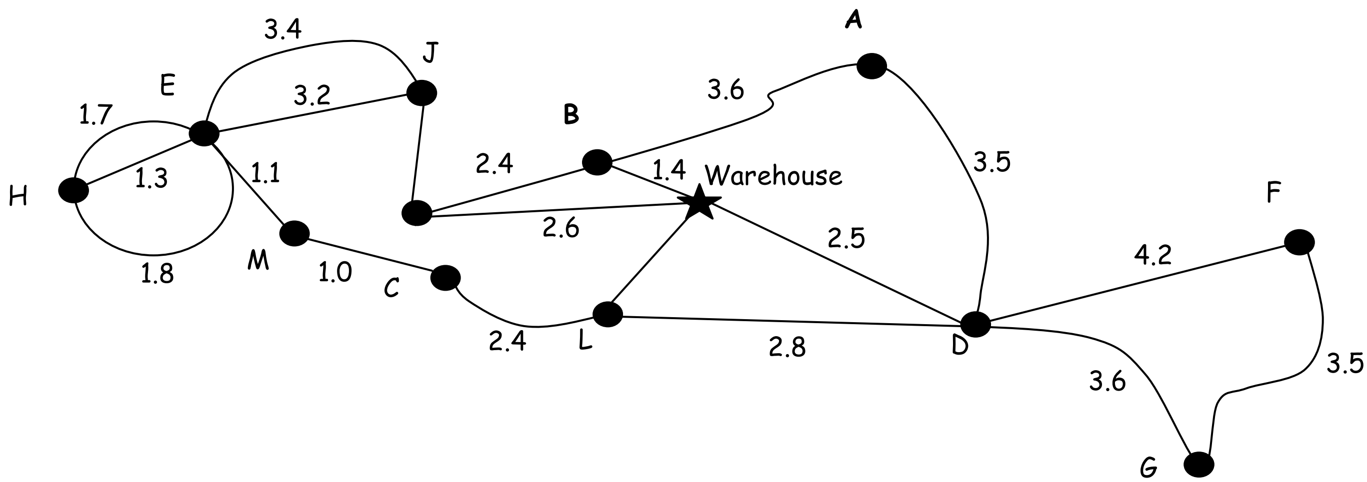
19. The following illustration represents the five boroughs of New York City and how they are connected by major bridges and tunnels.



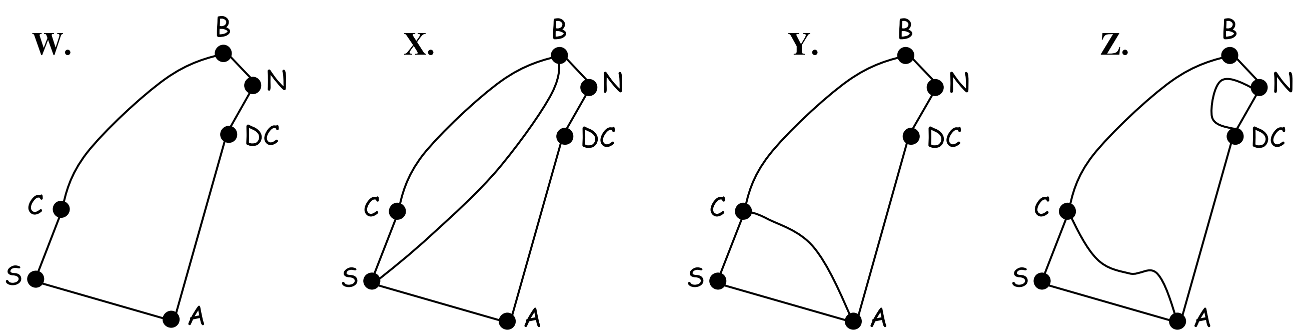
Is it possible to cross each bridge or tunnel exactly once and end up in the borough where you started? Is it possible to cross each bridge or tunnel exactly once? Explain your answer using what you know about networks.

20. The delivery people for Sleep Tight Beds must make deliveries today at each of the locations identified below by an upper case letter. They must begin and end their route at the warehouse and they want to follow the shortest possible route. All distances shown are in miles.

What path should they follow?

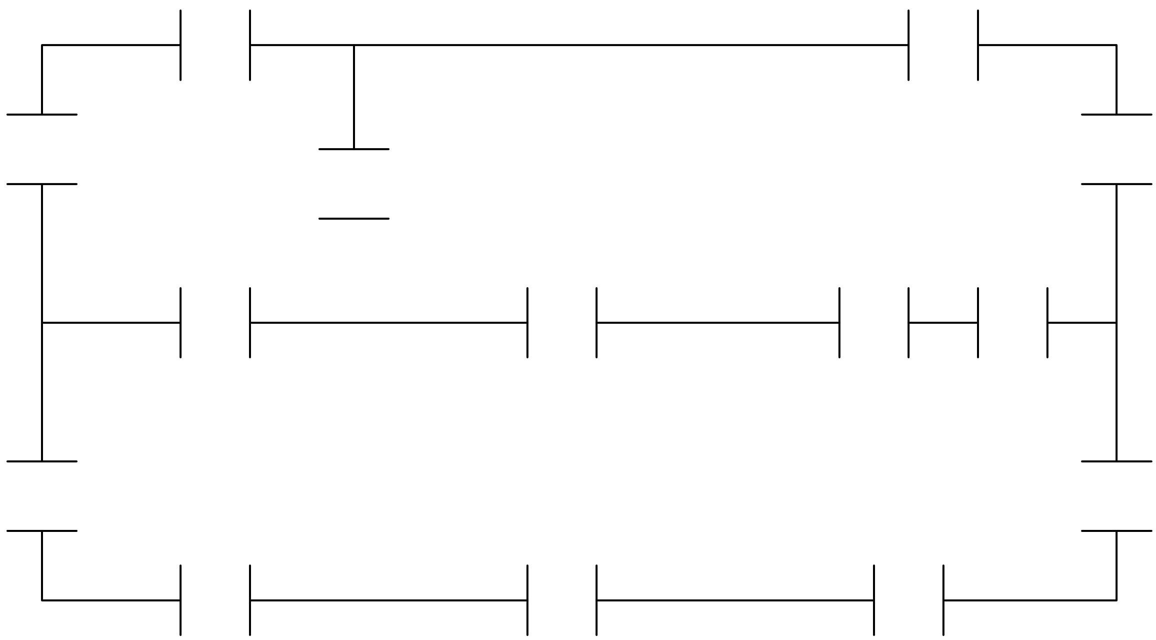


21. A flight begins in Chicago © and must make at least one stop in Boston (B), Washington D.C. (DC), Atlanta (A), Newark (N), and St. Louis (S), before returning to Chicago. Which path through each city does NOT represent a traversable network?



Use the floor plan of the house illustrated below to complete exercise 22.

22. Is it possible to walk through the house and go through each doorway exactly once? If it is possible, draw the path the person could follow. If it is not possible, explain why it is not possible.



23. Mackenzie and her family are planning a trip to Fantasy Kingdom Amusement Park. They wish to visit each area of the park and not have to pass through it again on their way to any other area. They also want to use each of the six rides or walkways once. Mackenzie believes it is impossible, while her brother Nicholas believes that it is possible. Who is correct and why?